

# Magneto-electric coupling in a two-dimensional ballistic Josephson junction with in-plane magnetic texture

François Konschelle

JARA-Institute for Quantum Information, RWTH Aachen University, D-52074 Aachen, Germany\*

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We study a Josephson junction made with a spin-textured bridge, when both Rashba and Zeeman interactions combine to generate a magneto-electric coupling between the superconducting current and the in-plane magnetic texture in the normal region. In particular, we unambiguously obtain the so-called anomalous current-phase relation  $j = j_c \sin \varphi + j_a \cos \varphi$  in a two-dimensional ballistic Josephson junction close to the critical temperature of the heterostructure, when an anomalous current  $j_a \neq 0$  subsists even at zero phase-difference between the superconductors, and is responsible for the coupling between the magnetic and electric degrees of freedom of the junction. The anomalous magneto-electric current is due to the combination of the chirality of the propagating modes and the anisotropy of the in-plane magnetic texture.

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Thanks to the recent progresses in heterostructures fabrication, it becomes possible to study the Josephson effect in the presence of strong spin-orbit effects [1–3]. Then we can envision possible developments of spin-textured (ST) Josephson systems, when the spin-orbit effect combines with the well-established superconducting-ferromagnet (S/F) proximity effect [4, 5]. This will certainly open the way to interesting discoveries, ranging from the fundamental question of the macroscopic quantum effects under exotic conditions [6, 7], to topological order useful for quantum information [8], passing through the coherent manipulation of mesoscopic circuits [9].

As an example of actual interest, the anomalous current-phase relation  $j = j_c \sin \varphi + j_a \cos \varphi$  (*i.e.*  $j = j_0 \sin(\varphi + \varphi_0)$ ) attracted some attentions over the past years [10–20]. Such  $\varphi_0$ -junctions exhibit a non vanishing super-current even at zero phase-difference between the two superconductors  $j(\varphi = 0) \neq 0$ , called an anomalous current  $j_a$ . Their possible applications are numerous: they produce a self-sustained flux when embedded in a SQUID geometry [10], they can act as some phase batteries in coherent circuits [11], they present a current asymmetry and act as a supercurrent rectifier [15], ...

A Josephson junction (JJ) with an anomalous current necessarily breaks the time-reversal symmetry since then  $j(\varphi) \neq -j(-\varphi)$  [21]. Then it seems natural to look for a  $\varphi_0$ -junction in S/F/S systems [4, 5]. So far, single junctions produced only  $\varphi_0 = \{0, \pi\}$ , the so-called  $\pi$ -JJ. In contrary, a parallel (0- $\pi$ )-JJ demonstrated the  $\varphi_0$ -behavior, with an extrinsic anomalous current induced by an external magnetic field [12, and references therein]. In a try to obtain an intrinsic anomalous current the spin-orbit interaction enters the stage: since it allows a manipulation of the critical current in two-dimensional JJ [22, 23], the hope is to obtain a JJ with a  $\varphi_0$ -phase-shift adjustable by an external gate voltage. Then some JJs

with both Zeeman and Rashba interactions have been intensively investigated over the past few years [13–20, and references therein].

In particular, Buzdin proposed a simple model for a S/ST/S Josephson junction and found a remarkable expression for the phase-shift  $\varphi_0 \propto (\mathbf{h} \times \mathbf{n}) \cdot \nabla \varphi$ , allowing a direct coupling between magnetization and super-current in the junction [14]. Such a coupling was found using symmetry arguments (see *e.g.* [24] for a detailed derivation of the associated Ginzburg-Landau functional) and can intrinsically be tuned by adjusting the paramagnetic interaction  $\mathbf{h}$  and/or the gate voltage, modifying the spin-orbit vector  $\mathbf{n}$ . Then the super-current allows the generation of the magnetization dynamics through the gradient of the superconducting phase  $\nabla \varphi$  (the direction of the junction, say) and back-action effect as well

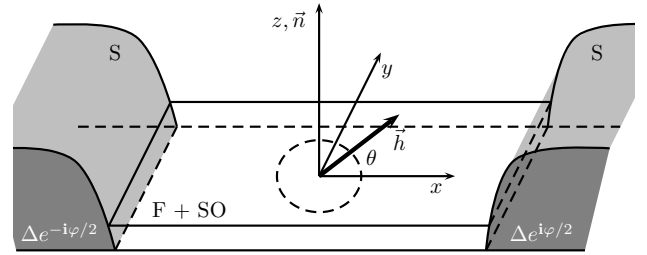


Figure 1: Schematic of the junction studied in this paper. A two-dimensional gas where the spin-orbit (along the  $z$ -axis, represented by  $\mathbf{n}$ ) and paramagnetic (in the  $xy$ -plane, represented by  $\mathbf{h}$  and angle  $\theta$ ) interactions compete (white region) is sandwiched between two superconducting banks (gray regions). The phase difference between the two superconducting electrodes is  $\varphi$ , and the gap parameters  $\Delta$  are constant and equals in the superconductors. Note that the exchange interaction could be either due to a magnetic interaction inside the junction, or to an applied magnetic field in the  $xy$ -plane.

[25].

Despite its interesting consequences, the magneto-electric origin of the anomalous current in ST-JJ has never been unambiguously clarified in numerical analysis [15–20]. In particular, most of the simple models built-on from the numerical investigations necessitate a mixing of chiral mesoscopic channels for the derivation of an anomalous current  $j_a$  [15, 16, 19, 20], hindering its magneto-electric origins.

Here, we discuss the presence of magneto-electric effect in Josephson physics. We use our recently proposed gauge-covariant transport formalism [26, 27] to exhibit the anomalous  $\mathbf{j} = j_c \sin \varphi + j_a \cos \varphi$  current-phase relation in a S/ST/S-JJ where the normal region combines Zeeman and Rashba interactions, with  $j_a$  in (16) and (21). We establish some generic expressions for  $j_c$  and  $j_a$  in (15) and (16) for a ballistic 2D-JJ in the presence of a non-trivial spin-field. This gauge-field accounts for the chirality induced by the spin-orbit interaction. In addition, the anisotropy of the Fermi surface, due to a paramagnetic effect in the plane of the junction, leads to a geometric coupling between the chiral propagation and the direction of the magnetic interaction. We then show that these two minimal conditions (anisotropic chirality and time-reversal-symmetry breaking) lead to a geometric coupling  $\varphi_0 \propto (\mathbf{h} \times \mathbf{n}) \cdot \nabla \varphi$  (see (22)), independent of the microscopic details: those are not relevant in our quasi-classical formalism indeed.

We first discuss some of the difficulties to obtain simple solutions of the complicated problem of a ST-JJ. For instance, if we suppose a free-electron gas with spin-orbit, ferromagnetic and two-body BCS interaction in a 1D configuration, there is no effect associated with the spin-texture. To see this in more details, we write

$$H = \int dx \left[ \Psi^\dagger H_0 \Psi + \frac{V(x)}{2} (\Psi^\dagger (\mathbf{i}\sigma_2) \Psi)^\dagger (\Psi^\dagger (\mathbf{i}\sigma_2) \Psi) \right]$$

$$H_0 = \frac{p_x^2}{2m} + \frac{p_y^2}{2m} - \mu + \mathbf{h} \cdot \boldsymbol{\sigma} + v_{so}(\mathbf{n} \times \mathbf{p}) \cdot \boldsymbol{\sigma} \quad (1)$$

for the Hamiltonian of the system, with  $\Psi^t = (\Psi_\uparrow \ \Psi_\downarrow)$  a spinor of fermionic annihilation operators,  $\sigma_i$  the Pauli matrices in the spin-space,  $p_{x,y}$  the electron momentum in the  $x, y$ -direction,  $m$  the electron (effective) mass,  $\mu$  the chemical potential,  $\mathbf{h} = h(\cos \theta, \sin \theta, 0)$  the exchange field in the  $xy$ -plane,  $v_{so}$  the Rashba interaction strength of direction  $\mathbf{n}$  (a unit vector along the  $z$ -axis), and  $V(x)$  the strength of the two-body interaction, giving rise to superconductivity in some regions of space. Note in passing that the Hamiltonian (1) somehow describes the dual setup than for Majorana modes (see *e.g.* [28, 29, and references therein] for some studies of the Josephson effect in the Majorana geometry), since in the later case the Zeeman interaction is along the  $z$ -axis, whereas we choose  $\mathbf{h}$  to lie in the  $xy$ -plane instead.

Now we reduce the problem to 1D, say the  $x$ -axis of the junction, and we choose  $\theta = \pi/2$  in order to maximize  $\varphi_0$ . Then, neglecting the  $p_y$  component, one can show that the transformation  $\Psi \rightarrow R\Psi$  with  $R = \exp[-\mathbf{i}\sigma_2 \int mv_{so}dx/\hbar]$  removes the spin-orbit interaction without affecting the singlet-pairing term in the BCS Hamiltonian. Thus under the above hypothesis, the model never exhibits any anomalous current; of course the model can still be used to study all the phenomenology associated with the S/F proximity effect [4, 5].

So in order to establish a magneto-electric coupling, we have to discard one of our hypothesis: either the junction should be explicitly two-dimensional, or the band dispersion should not be quadratic. Buzdin already discussed the second option [14]: when the dispersion is not quadratic, the ST might not be gauge trivial, even in 1D problems. Most of the numerical works in contrary discussed the first option; then, in order for some anomalous current to exist when the band dispersion is quadratic, a pinch-off of the structure seems to be necessary [15, 16, 19, 20]. Indeed, the anomalous current-phase relation is picturesquely supposed to come from the mixing of different channels along the junction in these studies. This additional difficulty renders difficult the establishment of the geometric structure of the anomalous current  $j_a$ .

In the following, we study a ST-JJ, when the normal part consists in a two-dimensional material lying in the  $xy$ -plane in the region  $-L/2 \leq x \leq L/2$ , with both Zeeman and Rashba interactions, and sandwiched between two conventional ( $s$ -wave) superconductors in the regions  $x \leq -L/2$  and  $x \geq L/2$ , see Fig. 1. Following the approach in [26, 27], the gauge-covariant transport equation associated with the Hamiltonian (1) reads, in the clean limit

$$-\mathbf{i}\hbar(\mathbf{v}_F \cdot \nabla) \mathbf{g} + [\mathbf{h} + \boldsymbol{\Delta}, \mathbf{g}] - \mathbf{i}mv_{so}^2 \partial_\phi \{\sigma_3, \mathbf{g}\} = 0 \quad (2)$$

where  $\mathbf{v}_F = v_F(\cos \phi, \sin \phi)$  is the projection of the Fermi velocity on the  $(x, y)$ -directions, respectively. We use the parameterisation

$$\mathbf{g} = \begin{pmatrix} g & -f \\ f^\dagger & g^\dagger \end{pmatrix}; \quad g = \begin{pmatrix} g_\uparrow & g_+ \\ g_- & g_\downarrow \end{pmatrix} \quad (3)$$

in the particle-hole and spin spaces, respectively, and the same parameterisation is used for the  $f, f^\dagger$  and  $g^\dagger$  sectors. Also,  $\mathbf{h} = \mathbf{h}_0 + \mathbf{h}_Z + \mathbf{h}_A$  where

$$\begin{aligned} \mathbf{h}_0 &= \hbar\omega\tau_3 \\ \mathbf{h}_Z &= h(\cos \theta\sigma_1 + \sin \theta\sigma_2)\tau_3 = (\mathbf{h} \cdot \boldsymbol{\sigma})\tau_3 \\ \mathbf{h}_A &= p_F v_{so}(\sin \phi\sigma_1 - \cos \phi\sigma_2) = h_A \hat{1} \end{aligned} \quad (4)$$

represent the different interactions. The matrix  $\boldsymbol{\Delta} = \Delta e^{\mathbf{i}\varphi}\tau_+ - \Delta e^{-\mathbf{i}\varphi}\tau_-$  represents the gap parameter with  $\varphi$  the superconducting phase-difference across the junction,

the  $\tau_i$  are Pauli matrices describing the particle-hole degree of freedom with  $\tau_{\pm} = (\tau_1 \pm i\tau_2)/2$ . We further suppose that the gap parameters and the phases are constant in the superconducting regions, and vanish in the normal regions.

We next recognize in (4) the free-particle term  $\mathbf{h}_0$ , the Zeeman  $\mathbf{h}_Z$  and the spin-orbit  $\mathbf{h}_A = \mathbf{v}_F \cdot \mathbf{A}$  interactions, linearised in the proximity of the Fermi surface. More importantly, the Fermi surface is supposed to be a single circle with a constant radius  $p_F$  in momentum space, parameterised by the angle  $\phi$  in (2). Indeed, the main advantage of the gauge-covariant formulation adapted to the transport formalism is to consider a free-electron gas, and to discuss the spin-orbit interaction as a non-Abelian gauge-potential  $\mathbf{A} = mv_{so}(-\sigma_y, \sigma_x, 0)/\hbar$  [26, 27]. It results a non-trivial gauge field  $F_{xy} = \partial_x A_y - \partial_y A_x + i[A_x, A_y] \propto mv_{so}^2 \sigma_3$ , leading to the last term in (2). This term takes into account the difference in angular-momentum between the two spinful sectors, see Fig. 2, and thus it is responsible for the chirality of the quasi-classical trajectories. An other important ingredient of the model is the Zeeman interaction  $\mathbf{h}_Z$  in (4). This interaction could be treated as a gauge-field as well [27], but here we suppose instead that the Fermi energy  $E_F \gg \hbar$ , and we treat  $\mathbf{h}_Z$  as a usual potential in the quasi-classic approximation [26]. Due to the presence of the ST, the genuine Fermi surface given by the spectrum of  $H_0$  in (1) is deformed, see Fig. 2. In our gauge-covariant transport formalism, in contrary, the Fermi surface is always supposed circular, and the anisotropy is taken into account as the combining effect of the chirality in the presence of the magnetic interaction.

We want to establish the current density flowing through a two-dimensional S/ST/S-JJ (see *e.g.* [27])

$$j = i\pi e N_0 v_F \int \frac{d\hbar\omega}{2\pi} \int_0^{2\pi} \frac{d\phi}{2\pi} \cos \phi \text{Tr} \{ \tau_3 \mathbf{g} \} \quad (5)$$

as a function of the exchange field, spin-orbit strength and phase difference across the JJ. The current being conserved, we can evaluate it at any position along the  $x$ -axis ; hereafter we choose the position  $x = 0$  in the middle of the normal region.

One can show that the system of equations (2) reduces to the form

$$i \frac{d\mathbf{g}}{ds} = \frac{L}{\hbar v_F} [\mathbf{h} + \mathbf{\Delta}, \mathbf{g}(s)] \quad (6)$$

where  $\mathbf{g}(s)$  is a short-hand notation for  $\mathbf{g}(z_\zeta, w_\zeta, \phi_\zeta)$  and with the characteristics

$$\begin{aligned} \phi_\zeta(s) &= \phi + 2\zeta s \\ x_\zeta(s) &= -\frac{L}{2\zeta} \sin \phi + \frac{L}{2\zeta} \sin(2\zeta s + \phi) \\ y_\zeta(s) &= \frac{L}{2\zeta} \cos \phi - \frac{L}{2\zeta} \cos(2\zeta s + \phi) \end{aligned} \quad (7)$$

with  $\zeta$  a parameter due to the non-trivial gauge-field. This important parameter exhibits the hallmark of the spin-orbit effect in term of the quasi-classic trajectories. From (2) we realize that the only components affected by the anti-commutator are the spinful components  $g_\uparrow, g_\downarrow, f_\uparrow, \dots$ . The associated characteristics are circular trajectories characterized by the parameter  $\zeta = \pm(L/\xi_{so})(v_{so}/v_F)$ , with  $\xi_{so} = \hbar/mv_{so}$  a spin-orbit length. So we see that, in addition to being bent, the trajectories of the spinful components are also spin-polarised, reminiscent of the chiral propagation of particles in systems with a strong spin-orbit interaction. In contrary, the characteristic trajectories associated with the singlet components  $g_+, g_-, f_+, \dots$  are not affected by the chiral symmetry, and thus propagate along straight lines  $\zeta = 0$  in (7). Importantly, the limit  $\zeta \rightarrow 0$  is not perturbative in (6): a non-zero  $\zeta$  modifies the topology of the trajectories, which become circles for the spinful components. The strategy to obtain the quasi-classic Green functions in (6) is thus clear: we first resolve the sys-

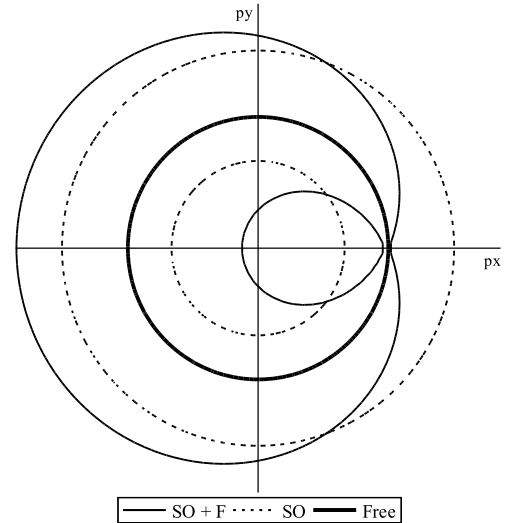


Figure 2: Fermi surfaces for non-interacting particles with and without magnetic interaction, described by the Hamiltonian (1). A free gas with a Rashba interaction leads to two spin-sub-bands (dashed lines, labelled SO). The addition of the Zeeman interactions leads to some anisotropy between these two surfaces (the two plain curves, labelled SO + F). In contrary, the gauge-covariant quasi-classic method simply requires a free-electron gas (*i.e.* a quadratic dispersion relation, the bold circle labelled Free) whereas the spin-orbit effect is treated as a gauge-potential, leading to a non-trivial gauge-field (see the text). Remarkably, the direction of maximum anisotropy is given by  $\mathbf{h} \times \mathbf{n}$ , see Eq.(1). In the plot, the Zeeman interaction is supposed really strong and along the  $y$ -axis, such that the anisotropy is mainly along the  $x$ -axis. The gauge-covariant quasi-classic expansion in principle breaks long before reaching such a large anisotropy and Fermi surface splitting, intended for illustration purpose only.

tem (6) for a general  $s$ , and afterwards we substitute the characteristics (7) according to

$$g = \begin{pmatrix} g_{\uparrow}(s_{\zeta=Lv_{\text{so}}/\xi_{\text{so}}v_F}) & g_{+}(s_{\zeta=0}) \\ g_{-}(s_{\zeta=0}) & g_{\downarrow}(s_{\zeta=-Lv_{\text{so}}/\xi_{\text{so}}v_F}) \end{pmatrix} \quad (8)$$

and the same substitution for the other spin-matrices  $f$ ,  $f^{\dagger}$  and  $g^{\dagger}$ , where  $s_{\zeta}$  is the inverse of the system (7) depending parametrically on  $\zeta$  (see (17) below).

Under the form (6), the transport equation admits an Ansatz

$$\mathbf{g}(s) = \mathbf{u}(s) \mathbf{g}_0 \mathbf{u}^{-1}(s) + \mathbf{g}_{\infty} \quad (9)$$

with  $\mathbf{g}_0$  and  $\mathbf{g}_{\infty}$  some constant matrices [30, 31]. Such an Ansatz allows to resolve Eq.(6) in the form of the commutator relation  $[\mathbf{h} + \mathbf{\Delta}, \mathbf{g}_{\infty}] = 0$  in addition to the Schrödinger-like equation

$$\mathbf{i} \frac{d\mathbf{u}}{ds} = \frac{L}{\hbar v_F} (\mathbf{h} + \mathbf{\Delta}) \mathbf{u}(s) \quad (10)$$

solved directly in a matrix form for the propagation-like operator  $\mathbf{u}$  instead of resolving each components of the  $\mathbf{g}$  matrix individually. Also, an expression like (10) can be perturbatively treated as usual with a Schrödinger-type equation [32].

In the superconductors with a  $s$ -independent  $\mathbf{\Delta}$ , and also  $\mathbf{h}_Z = \mathbf{h}_A = 0$ , the gauge-field disappears, so the trajectories are some straight lines given by  $\zeta = 0$  in (7). Injecting the Ansatz (9) into (6), one has  $[\mathbf{h}_0 + \mathbf{\Delta}, \mathbf{g}_{\infty}] = 0$  which has for solution  $\mathbf{g}_{\infty} = N_g (\mathbf{h}_0 + \mathbf{\Delta})$  with the constant  $N_g$  such that  $\mathbf{g}_{\infty}^2 = 1$ . It is important to note that the normalization property is preserved due to the absence of the gauge-field in the superconductor, see *e.g.* [27]. Defining for commodity  $\cos \eta = \hbar\omega/\Delta$  and  $\xi = \hbar v_F/\Delta$  for the superconducting coherence length, one has

$$\begin{aligned} \mathbf{g}\left(x \leq -\frac{L}{2}\right) &= e^{-i\tau_3 \frac{\varphi}{4}} [\mathbf{S}_{+} g_1 \tau_{+} \mathbf{S}_{+}^{-1} + \mathbf{g}_{\infty}^0] e^{i\tau_3 \frac{\varphi}{4}} \\ \mathbf{g}\left(x \geq \frac{L}{2}\right) &= e^{i\tau_3 \frac{\varphi}{4}} [\mathbf{S}_{-} g_2 \tau_{-} \mathbf{S}_{-}^{-1} + \mathbf{g}_{\infty}^0] e^{-i\tau_3 \frac{\varphi}{4}} \end{aligned} \quad (11)$$

in the two superconductors, with  $\mathbf{S}_{\pm} = (\tau_{\downarrow} - e^{i\eta} \tau_{\uparrow} + \tau_{-} - e^{-i\eta} \tau_{+}) e^{\tau_3 \sin \eta (x \pm L/2)/\xi}$  and  $\mathbf{g}_{\infty}^0 = \mathbf{i}(\cos \eta \tau_3 + \mathbf{i} \tau_2) / \sin \eta$ , with  $\tau_{\uparrow, \downarrow} = (1 \pm \tau_3)/2$ . The two matrices  $g_{1,2}$  are now constant matrices in the spin-space given by boundary conditions. In (11) we see why we needed the constant matrix  $\mathbf{g}_{\infty}$ : it accounts for the bulk properties of superconductivity, on top of which some evanescent waves are localized close to the interfaces.

In the normal part, we suppose  $\mathbf{\Delta} = 0$ , and thus  $\mathbf{g}_{\infty} = 0$  in the Ansatz (9) since we expect to find propagating modes instead of evanescent ones. Still, the term  $\mathbf{h}_A$  is  $s$ -dependent, since it depends on the angle  $\phi \equiv \phi_{\zeta}(s)$ . We have thus  $\mathbf{g}(s) = \mathbf{u}(s) \mathbf{g}_0 \mathbf{u}^{\dagger}(s)$  with the propagation

operator

$$\mathbf{u} = e^{-\mathbf{i} \frac{\omega L}{v_F} s \tau_3} \begin{pmatrix} u_{+} & 0 \\ 0 & u_{-} \end{pmatrix} \quad (12)$$

$$\mathbf{i} \frac{du_{\pm}}{ds} = \frac{L}{\hbar v_F} (\pm (\mathbf{h} \cdot \boldsymbol{\sigma}) + h_A) u_{\pm}(s) \quad (13)$$

in the particle-hole space. We do not need an explicit and/or perturbative form for  $u_{\pm}(s)$  until Eq.(19) below.

Next step is to use rigid boundary conditions to obtain the generic form of the particle-sector spin-matrix  $g_0$  in the normal region. Thanks to the decoupling between particles and holes in the normal region (see (12)), one can write the formula

$$j = j_c \sin \varphi + j_a \cos \varphi \quad (14)$$

$$\begin{aligned} \frac{j_c}{j_{\Delta}} &= -2 \int_{-\pi/2}^{\pi/2} d\phi \cos \phi \times \\ &\sum_{\Sigma} e^{\Sigma L (s_L^{\Sigma} - s_R^{\Sigma}) / 2\xi_T} \text{Tr} \left\{ U_L \cdot U_R^{\dagger} \right\} \end{aligned} \quad (15)$$

$$\begin{aligned} \frac{j_a}{j_{\Delta}} &= \frac{\mathbf{i}}{2} \int_{-\pi/2}^{\pi/2} d\phi \cos \phi \times \\ &\sum_{\Sigma} e^{\Sigma L (s_L^{\Sigma} - s_R^{\Sigma}) / 2\xi_T} \text{Tr} \left[ \left\{ U_R, U_L^{\dagger} \right\} - \left\{ U_R^{\dagger}, U_L \right\} \right] \end{aligned} \quad (16)$$

close to the critical temperature  $T_c$  of the junction, where we defined  $U = u_{+}^{\dagger}(s) u_{-}(s)$ ,  $U_{L,R} = U(s_{L,R}^{\Sigma})$  and  $j_{\Delta} = e N_0 v_F \Delta^2 / \pi^2 k_B T_c$ . To find (14) we transformed the expressions for  $\mathbf{g}$  both in the superconductors and in the normal region to the Matsubara representation  $\omega \rightarrow i\omega_n$ ,  $\hbar\omega_n \approx \pi k_B T_c (2n+1)$ , we expanded in the small  $\Delta/k_B T_c$  parameter, and we supposed  $L/\xi_T \gg 1$  with the thermal length  $\xi_T = \hbar v_F / 4\pi k_B T_c$ . We also defined  $s_{L,R}^{\Sigma} = s(x = \mp L/2, \Sigma)$  as the expressions for the  $s$ -parameter at the boundaries between the superconductors and the spin-textured regions and depending in the direction of the propagation  $\Sigma = \pm 1$ .

Eq.(14) is the main result of this letter. It gives a generic expression for the first harmonics of the Josephson current in a clean junction. Especially, it contains an explicit expression for the anomalous current  $j_a$ . We see in the structure of the  $U_{L,R}$  matrix that it corresponds to the propagation of an electron (hole) from  $s = 0$  (the middle of the junction in our parametrisation) toward the point  $s_{L,R}$  corresponding to the superconducting interface, where it is reflected back as a hole (electron) to  $s = 0$ . So the product  $U_L U_R^{\dagger}$  represents the entire exploration of the particles inside the normal region, giving rise to the Andreev modes in the junction which in turn transport the current in (15). In contrary, the anomalous



current is due to some interferences between the Andreev modes in (16). Clearly,  $j_a$  vanishes for non magnetic bridges, since  $u_{\pm} = 1$  in that case: to break the time-reversal symmetry is mandatory to obtain an anomalous current.

To go further one needs the expression of  $s(x_{\zeta})$ , which is given perturbatively as

$$s \approx \Sigma \left( \frac{x/L}{\cos \phi} + \left( \frac{x}{L} \right)^2 \frac{\sin \phi}{\cos^3 \phi} \zeta + \dots \right) \quad (17)$$

from (7) in the limit  $v_{so} \ll v_F$ . We see how the small chirality  $\zeta = (0, \pm L v_{so} / \xi_{so} v_F)$  affects the boundary conditions in the above expression. We then expand the expressions (15) and (16) for a small spin-orbit interaction. We obtain for instance ( $j_c$  does not contain any signature of the spin-orbit effect at first order)

$$\begin{aligned} \frac{j_a}{j_{\Delta}} = & -\frac{L^2}{\xi_{so} \xi_f} \frac{v_{so}}{v_F} \int_{-\pi/2}^{\pi/2} d\phi \frac{\sin \phi}{\cos^2 \phi} \times \\ & e^{-L\tilde{s}/\xi_T} \text{Tr} \left[ \sigma_3 \left\{ u_+^{\dagger} \left( \hat{\mathbf{h}} \cdot \boldsymbol{\sigma} \right) u_- , \bar{u}_-^{\dagger} \bar{u}_+ \right\} \right. \\ & \left. - \sigma_3 \left\{ u_+^{\dagger} u_- , \bar{u}_-^{\dagger} \left( \hat{\mathbf{h}} \cdot \boldsymbol{\sigma} \right) \bar{u}_+ \right\} \right]_{\zeta=0} \quad (18) \end{aligned}$$

where  $\tilde{s} = 1/2 \cos \phi$ ,  $\hat{\mathbf{h}} = \mathbf{h}/h$ ,  $\xi_f = \hbar v_F / h$  the ferromagnetic coherence length, and we defined  $u_{\pm} \equiv u_{\pm}(\tilde{s})$  and  $\bar{u}_{\pm} \equiv u_{\pm}(-\tilde{s})$  for the sake of compactness. The presence of the  $\sigma_3$  matrix in (18) is due to the chirality in (8). The anomalous current (18) vanishes either for a vanishing exchange  $h \rightarrow 0$  or a spin-orbit  $v_{so} \rightarrow 0$  interaction. Since the spin-orbit effect induces a chiral propagation along the junction, whereas the magnetic interaction induces an anisotropic Fermi surface (see Fig. 2) in addition to the breaking of the time-reversal symmetry, these two conditions are minimal for the existence of the anomalous current.

To obtain explicit values for  $j_a$ , one has to solve (13). This can be done in perturbation, supposing  $h \gg p_F v_{so}$ . Then we have

$$\begin{aligned} u_{\pm}(s) = & e^{\mp i L \hat{\mathbf{h}} \cdot \boldsymbol{\sigma} s / \xi_f} \times \\ & \left( 1 - \frac{iL}{\hbar v_F} \int_0^s e^{\pm i L \hat{\mathbf{h}} \cdot \boldsymbol{\sigma} s_1 / \xi_f} h_A(s_1) e^{\mp i L \hat{\mathbf{h}} \cdot \boldsymbol{\sigma} s_1 / \xi_f} ds_1 \right) \quad (19) \end{aligned}$$

at first order in  $L/\xi_{so}$ . We verified that when  $v_{so}/v_F = 0$ , then  $j_a = 0$  and  $j_c \propto j_{\Delta} \cos(2L/\xi_f)$ , as usual for a pure S/F/S system [4], which further reduces to the S/N/S case when the exchange field disappears [21]. Also, even for  $v_{so}/v_F \neq 0$  but when  $\mathbf{h}$  is along the  $z$ -axis, then  $j_a = 0$  in (18). So to break the time-reversal symmetry in a chiral system is not sufficient to obtain  $j_a \neq 0$ : the anisotropy of the Fermi surface is also mandatory (see Fig. 2).

We finally inject (19) into (18) and (15) and we obtain

$$\frac{j_c}{j_{\Delta}} = -\sqrt{8\pi} \frac{e^{-L/2\xi_T}}{\sqrt{L/2\xi_T}} \cos \frac{2L}{\xi_f} \quad (20)$$

$$\frac{j_a}{j_{\Delta}} = \sqrt{2\pi} \frac{L^3}{\xi_f \xi_{so}^2} \frac{v_{so}}{v_F} \sin \theta \frac{e^{-L/2\xi_T}}{(L/2\xi_T)^{3/2}} \sin \frac{L}{\xi_f} \quad (21)$$

up to the first non-trivial term in the approximation  $h \gg p_F v_{so}$ . Note that (14) can also be written  $j \approx j_c \sin(\varphi - \varphi_0)$  at the first non-trivial order in  $v_{so}/v_F$ , with a magneto-electric phase-shift

$$\tan \varphi_0 = \frac{1}{2} \frac{L^2 \xi_T}{\xi_{so}^2 \xi_f} \frac{v_{so}}{v_F} \sin \theta \frac{\sin L/\xi_f}{\cos 2L/\xi_f} \quad (22)$$

and thus presents some phase-kinks when the cosine term in (20) vanishes. These phase-kink are reminiscent of the transition between the 0- and  $\pi$ -phases in S/F/S-JJ [4].

A relation similar to (22) was obtained by Buzdin in the case of an interacting fermionic gas, see [14] and discussion after Eq.(1). We here recover his result in the case of a simpler Fermi liquid with non-trivial in-plane magnetic texture.

The current-phase relation (14) is remarkable. First, the critical-current  $j_c$  in (20) exhibits oscillations with respect to the length of the junction, as usual with S/F proximity effect [4]. In addition, a SQUID made with a S/ST/S-JJ exhibits a self-generated flux [10, 14], due to the anomalous current in (21). This flux can be quenched by changing the relative orientation of the exchange field with respect to the direction of the junction (the  $\sin \theta$  term in (21)), *e.g.* by the application of an external magnetic field in the plane of the junction. Also, the spin-orbit interaction  $v_{so}$  can be modified by the application of a gate-voltage, as demonstrated for 2D-JJ [22, 23], which in (14) also changes the strength of the anomalous current  $j_a$ , proportional to  $(v_{so}/v_F)^3$  in the limit  $h \gg p_F v_{so}$ . All these effects can be easily demonstrated using currently available nano-technologies. Note also the possibility to excite the magnetization using a voltage-biased S/ST/S-JJ, as shown in [25].

It is delicate to compare our results with the numerical ones [15–20], since our quasi-classic treatment in principle accounts for a large number of channels, whereas the numerical works focused essentially on a small number of channels. Note the exception [18], where an almost sinusoidal current-phase relation has been found numerically in the many-channels situation, in a form similar to (14). Also, to have chiral propagation has been widely acknowledged as a necessary condition for the obtention of a  $\varphi_0$ -junction [15, 16, 18–20]. Moreover the dependency of  $j_a$  with respect to the orientation of the magnetic effect has been discussed in a few occasions [15–19]. I hope that the present study may help understanding the geometric nature of the magneto-electric coupling in the anomalous

current (18). Also, I clearly showed that the presence of the quantum-point-contact is not a requirement for the anomalous current to exist (see also [13, 14, 17, 18]). In addition, the  $\varphi_0$ -behavior does not require neither a long junction nor an interacting gas to exist.

In conclusion, using a gauge-covariant transport formalism to take into account the spin and charge degrees of freedom of the Cooper pairs on equal footing, I showed that a magneto-electric coupling arises in spin-textured Josephson junctions. This magneto-electric effect is due to the chirality of the propagation modes inside the spin-textured region, its explicit geometric nature being due to the anisotropy of the Fermi surface induced by the breaking of the time-reversal-symmetry, see Fig. 2. More importantly, these two criteria (anisotropic chirality and time-reversal-symmetry breaking) are the minimal requirements for the existence of the magneto-electric coupling. I here established an anomalous current-phase relation  $j = j_0 \sin(\varphi - \varphi_0)$  (i.e.  $j = j_c \sin \varphi + j_a \cos \varphi$ , Eqs. (14)-(16)) in the proximity with the critical temperature of a ballistic junction, with  $\varphi_0 \propto (\mathbf{h} \times \mathbf{n}) \cdot \nabla \varphi$  a geometric term coupling the exchange field  $\mathbf{h}$  to the superconducting phase difference  $\varphi$  via the spin-orbit orientation  $\mathbf{n}$ , according to (22). Such a  $\varphi_0$ -phase-shift induces an anomalous current  $j_a \neq 0$ , Eq.(21). Note finally that similar results exist in the diffusive limit [33] and that a more detailed version of this work is in preparation.

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\* Electronic address: [konschelle@physik.rwth-aachen.de](mailto:konschelle@physik.rwth-aachen.de)

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